

1. A partial derivative of a function with two or more variables with respect to one variable. Treat all other variables as constant.

Find the partial derivative of the following function:

$$f(x, y, z) = \frac{x^4}{z^2} + 2x \ln(y) + y\sqrt{z}$$

- a.) With respect to x

$$f_x(x, y, z) = \frac{4x^3}{z^2} + 2 \ln(y)$$

- b.) With respect to y

$$f_y(x, y, z) = \frac{2x}{y} + z^{.5}$$

- c.) With respect to z

$$f_z(x, y, z) = \frac{-2x^4}{z^3} + \frac{y}{2z^{.5}}$$

2. The chain rule is used when dealing with a composite function $f(g(x))$ and the derivative is calculated as $f'(g(x)) * g'(x)$.

Find the derivative of the following functions:

- a.)

$$f(x) = \ln(x^2)$$

$$f'(x) = 2x * \frac{1}{x^2}$$

$$= \frac{2}{x}$$

- b.)

$$f(x) = e^{2x}$$

$$f'(x) = 2 * e^{2x}$$

3. Find the partial derivative of the following function:

$$f(x, y) = (3x + 4y^2)^2 + \ln(xy^2)$$

- a.) With respect to x

$$\begin{aligned} f_x(x, y) &= 2 * (3x + 4y^2) * 3 + y^2 * \frac{1}{xy^2} \\ &= 6(3x + 4y^2) + \frac{1}{x} \end{aligned}$$

- b.) With respect to y

$$\begin{aligned} f_y(x, y) &= 2 * (3x + 4y^2) * 8y + 2xy * \frac{1}{xy^2} \\ &= 16y(3x + 4y^2) + \frac{2}{y} \end{aligned}$$

4. What is the marginal utility of x for the following utility function?

$$U(x, y) = 5x^2y^3$$

$$MU_x = 10xy^3$$

5. What is the marginal utility of y for the following utility function?

$$U(x, y) = x^{.25}y^{.75}$$

$$\begin{aligned} MU_y &= .75x^{.25}y^{-.25} \\ &= .75 \frac{x^{.25}}{y^{.25}} \end{aligned}$$

6. Find the cross-partial derivative $f_{xy}(x, y)$ of the following function:

$$f(x, y) = 3x^2y$$

$$MU_x = 6xy$$

$$\frac{dMU_x}{dy} = 6x$$